Fluctuation forces
Lecture 5 of Statistical Mechanics and Fluctuations

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Last week

• Number and density fluctuations.

• The binomial, Poisson and Gaussian distributions.

• The van der Waals equation of state and critical fluctuations.
This week

- Forces due to fluctuations.
- The Casimir effect.
- van der Waals forces.
- Membranes.
The Casimir effect

Just as a molecule has zero-point vibrations, the vacuum has zero point fluctuations of the electromagnetic field.

Any electric field must vanish at the surface of an electrical conductor. If it didn’t, it would rearrange the electrons until it did.

A conductor changes the permitted fluctuations.

http://www.casimir.rl.ac.uk/default.htm
Electric fields

- Electrostatic force on an electrical charge is due to the field $F = qE$.

- The electric field drives electrical conduction, Ohm’s law is $J = \sigma E$.

- Light is an oscillating electromagnetic (electric+magnetic) field.

- A proper theory of the electric (or magnetic) field must be quantum, i.e. light is made of photons, and so there will be zero-point energy.
Field between conducting plates

electric field

conducting plates
Field between conducting plates

electric field

conducting plates
Field between conducting plates

This field would act on the electrons in the conducting plates, and rearrange them, and so is not stable.
Field between conducting plates
Fourier expansion
$$f(z) = \sum_{n}^{n} a_n g_n(z) = a_1 g_1(z) + a_2 g_2(z) + \ldots$$

Fourier expansion

Functions shifted for clarity.
Field between conducting plates
Field between conducting plates

electric field

conducting plates

z
Field between conducting plates

electric field

conducting plates
We can work out the force

The zero point energy of a harmonic oscillator is:

\[ E_0 = \frac{1}{2} \hbar \omega \]

We need to add up all the modes:

\[ E = \sum_{n} \frac{1}{2} \hbar \omega_n \]

The energy of the modes in the cavity is:

\[ \frac{E}{A} = -\frac{\hbar c \pi^2}{720a^3} \]

where \( a \) is the separation between the plates.
The Casimir effect

• An attractive force between metal plates, or other conductors.
• Due only to constraints on the allowed fluctuations, independent of material.
• Produced by quantum zero point fluctuations.
• Is it just the van der Waals interaction?
Electrons have zero point fluctuations too

Atoms are spherically symmetric on average, but have a fluctuating dipole moment. The attraction between these ‘instantaneous dipoles is usually modelled with a potential:

\[ u(R) = -\frac{C_6}{r^6} \]
Membrane fluctuation forces

Membrane fluctuation forces

Just as in the Casimir effect, the membranes’ fluctuations are constrained, and there is a force between them.

Membrane fluctuation forces

An elastically fluctuating membrane with randomly chosen oscillations:
Membrane fluctuation forces

Two identical membranes don’t fluctuate if they are far enough apart:
Membrane fluctuation forces
Membrane fluctuation forces
\langle h^2 \rangle \propto \log(L/a)

Membrane channels

Bastug, Gray-Weale, Patra and Kuyucak,
BIOPHYSICAL JOURNAL 90 (7): 2285-2296 APR 2006
Molecular ‘crowding’
Summary of fluctuation forces:

Membrane inclusions:
\[ V(R) = -kT \frac{A^2}{R^4} \times \frac{6}{\pi^2} \]

Membrane undulations:
\[ P = \frac{(kT)^2}{K_b a^3} \times \frac{3\pi^2}{64} \]

van der Waals:
\[ V(R) \propto -kT \frac{\alpha_1 \alpha_2}{R^6} \times \frac{\hbar \omega_0}{kT} \]

Casimir:
\[ \frac{E}{A} = -\frac{\hbar c \pi^2}{720a^3} \]